Broad-Band Impedance Matching into Dielectric-Filled Waveguides*

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Summary—**The problem of impedance matching between two waveguides filled with different dielectrics is discussed, and the conditions for broad-band matching are determined. Experimental results are presented for standard waveguides matched to guides filled with dielectrics having permittivities ϵ as high as $100\epsilon_0$. Present applications include matching devices for X-band coupled-cavity transmission masers which employ ruby and alumina sections ($\epsilon \approx 10\epsilon_0$). Future applications include matching devices for masers utilizing rutile (ϵ as high as $250\epsilon_0$).**

INTRODUCTION

IN THE DEVELOPMENT of microwave and millimeter-wave masers, it is necessary to provide means for matching from air-filled waveguides to guides filled with materials having high dielectric permittivities. For practical reasons, the device which makes the transition should be frequency-independent. The most common approach to this problem is to use a uniformly tapered transition; this consists of a tapered waveguide with a dielectric insert having reverse tapers.[1] As a result of preliminary work in which this approach was used, it was concluded that the desired performance could not be achieved reliably with a device of this kind.

These are the types of matching problems which arise in maser circuits. To the best of our knowledge, similar problems have not occurred in more conventional microwave circuits. However, a good matching technique is potentially applicable to a variety of miniaturized microwave circuits, including passive filters and transmission networks.

The maser material most commonly used to date is ruby, which has a tensor permittivity with a magnitude of about 10 ϵ_0. In the X-band ruby masers which have been developed at our laboratory, transitions for TE_{01} mode propagation from an air-filled guide to ruby- or alumina-filled guides are required at the input and output of the slow-wave structure. Another maser material which is finding increased application is rutile, doped either with chromium or iron. The permittivity of rutile is also a tensor quantity whose magnitude varies from 80 to 256, depending on the temperature and the angular orientation between the direction of the electric field and the optic axis.[2] The basic slow-wave structure for most rutile masers is simply a continuous waveguide filled with rutile. Since the external RF circuits are air-filled waveguides, the design of rutile masers requires the use of transitions capable of matching two sections of guide in which the permittivity may differ by a ratio of over 250 to 1.

This paper reports a method of obtaining a broad-band RF impedance match into waveguides filled with materials having high dielectric permittivities. An analysis is presented which shows that it is possible to obtain frequency independence at the junction of the two waveguides. Experimental verification is given for matching into alumina-filled and titania-filled waveguides, together with the design principles and construction details.

ANALYSIS

The analytical method takes advantage of the analogy between waveguides and transmission lines. The real part of the reflection at the junction of two waveguides is the same as that predicted at the junction of two transmission lines having the same impedance ratios.[3] The imaginary part of the reflection at the waveguide junction can be represented by a shunt susceptance in the transmission line circuit shown in Fig. 1. The impedance of a waveguide can be defined in many ways, all of which are consistent in that they are proportional to the wave impedance E_y/H_x. When the impedance definition is based on the power and maximum voltage in the waveguide,[4] we have for the fundamental mode

$$Z_1 = \frac{1}{Y_1} = 2\frac{b_1}{a_1}\left(\frac{E_y}{H_x}\right) = 2\frac{b_1}{a_1}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{\lambda_g}{\lambda_0}$$

$$= 2\frac{b_1}{a_1}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{1}{\sqrt{\epsilon_1 - \left(\frac{\lambda_0}{2a_1}\right)^2}}$$

$$Z_2 = \frac{1}{Y_2} = 2\frac{b_2}{a_2}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{1}{\sqrt{\epsilon_2 - \left(\frac{\lambda_0}{2a_2}\right)^2}},$$

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[1] E. S. Sabisky and H. J. Gerritsen, "Traveling-wave maser using chromium-doped rutile," PROC. IRE (*Correspondence*), vol. 49, pp. 1329–1330; August, 1961.

[2] E. S. Sabisky and H. J. Gerritsen, "Measurements of the dielectric constant of rutile (TiO_2) at microwave frequencies between 4.2° and 300°K," *J. Appl. Phys.*, vol. 33, p. 1450; April, 1962.

[3] G. L. Ragan, "Microwave Transmission Circuits," M.I.T., Rad. Lab. Ser., vol. 9, McGraw-Hill Book Co., Inc., New York, N. Y., p. 54; 1948.

[4] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., New York, N. Y., p. 319; 1943.

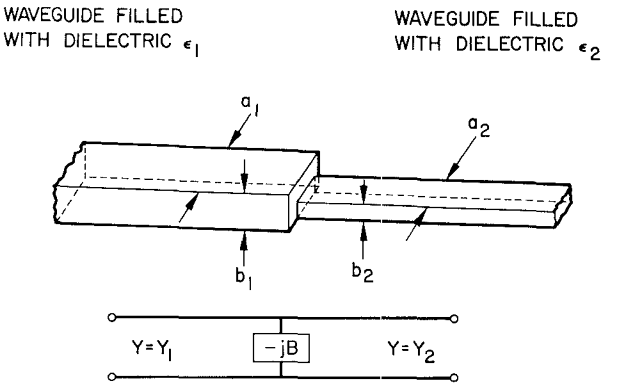

Fig. 1—Junction of two waveguides filled with different dielectrics.

where

$\lambda_0 \equiv$ free space wavelength
$a_1 \equiv$ width of first waveguide
$a_2 \equiv$ width of second waveguide
$b_1 \equiv$ height of first waveguide
$b_2 \equiv$ height of second waveguide
$\epsilon_1 \equiv$ dielectric permittivity of material in first guide
$\epsilon_2 \equiv$ dielectric permittivity of material in second guide
$\lambda_g \equiv$ waveguide wavelength
$-\sqrt{\mu_0/\epsilon_0} = 377\ \Omega$.

To match admittances of the two waveguides, we let $Y_1 = Y_2$; therefore,

$$\frac{a_1}{b_1}\sqrt{\epsilon_1 - \left(\frac{\lambda_0}{2a_1}\right)^2} = \frac{a_2}{b_2}\sqrt{\epsilon_2 - \left(\frac{\lambda_0}{2a_2}\right)^2}.$$

It is obvious from the above equation that for a given ϵ_1 and ϵ_2, the a and b waveguide dimensions may be adjusted to give a match at a particular frequency corresponding to λ_0.

To obtain a frequency-independent match, we must choose

$$b_1 = b_2$$

and

$$\frac{a_1}{a_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

Choosing $b_1 = b_2$ also simplifies the construction of the transition device. The criterion for determining b is that it must be small enough to prevent the TE_{11} mode from propagating and to preserve the TE_{01} mode. The ratio of b to a is $\frac{1}{2}$ for a waveguide filled with an isotropic medium. Since the b dimension does not change, the susceptance of the junction is therefore inductive, and it remains only to match out the reflections from the inductive susceptance $-jB$.

Anisotropy and Birefringence

When a waveguide is filled with a single-crystal paramagnetic material having tensor permittivity, care must be taken to avoid undesirable modes of propagation. For example, a uniaxial crystal with its c-axis oriented at an angle θ with the normal to the broad wall of the filled waveguide will have an effective dielectric constant ϵ_y in the E-plane of the waveguide, determined from the dielectric ellipsoid. Thus, we have for the fundamental mode:

$$\epsilon_y^{-1} = \epsilon_\parallel^{-1}\cos^2\theta + \epsilon_\perp^{-1}\sin^2\theta.$$

To prevent birefringence, the orthogonal mode cannot be allowed; therefore, the b/a ratio of the waveguide for the octave bandwidth condition is determined from

$$\frac{b}{a} < \frac{\epsilon_y}{2\epsilon_x},$$

where

$$\epsilon_x^{-1} \equiv \epsilon_\parallel^{-1}\sin^2\theta + \epsilon_\perp^{-1}\cos^2\theta.$$

Normally, a b/a ratio of $\frac{1}{2}$ is small enough to prevent birefringence in most crystals of interest (other than rutile) over octave bandwidths.

Design and Fabrication of a Transition from Air- to Dielectric-Filled Waveguides

In order to meet the frequency-independent condition, $b_1 = b_2$, the E-dimension of the air-filled waveguide is reduced by either a taper[5] or a series of steps[6] to the height of the dielectric-filled waveguide (Fig. 2). The air-filled waveguide is then joined directly to the dielectric-filled guide, whose width is determined by $a_2 = a_1/\sqrt{\epsilon/\epsilon_0}$. The inductive susceptance is matched by using a circular capacitive stub placed in the air-filled waveguide close to the junction. The optimum location of the stub was found experimentally to be a distance $a/2$ from the dielectric, and the optimum stub diameter was found to be 0.8a (see Fig. 3).

It is essential to obtain a perfect filling factor in the loaded waveguides since relatively minor flaws in the filling factor introduce large susceptances in the guide. One successful technique is to metallize the dielectric material with silver and then to electroform with silver or copper to a nominal thickness to strengthen the part and to permit soldering to appropriate flanges. The losses encountered in dielectric-filled waveguides vary widely, depending upon the quality of the dielectric and the metallizing. Silver-plated alumina has 0.1 db/in of loss at room temperature. Since the dielectric loss accounts for 0.03 db/in, it may be surmised that most of the loss is contributed by the conductive plating.

[5] K. Matsumaru, "Reflection coefficient of E-plane tapered waveguides," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 143–149; April, 1958.

[6] L. Young, "Tables for cascaded homogeneous quarter-wave transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 233–237; April, 1959.

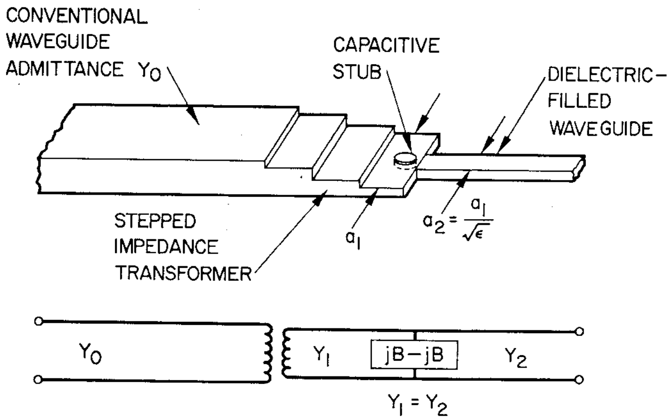

Fig. 2—Junction of a conventional waveguide with one filled with a dielectric.

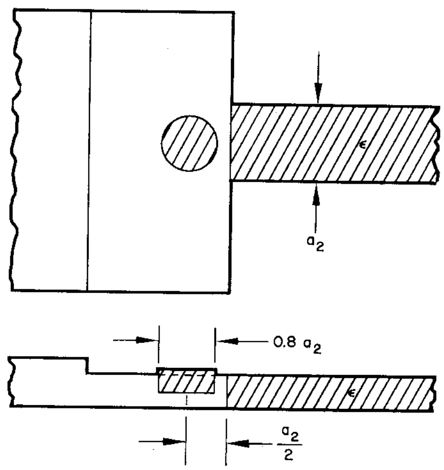

Fig. 3—Matching stub configuration.

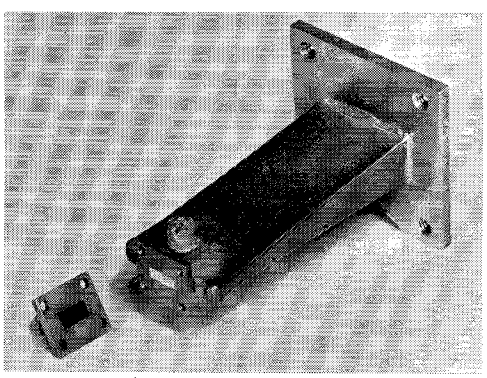

Fig. 4—Transition from an RG-52 to an alumina-filled waveguide and polyiron termination.

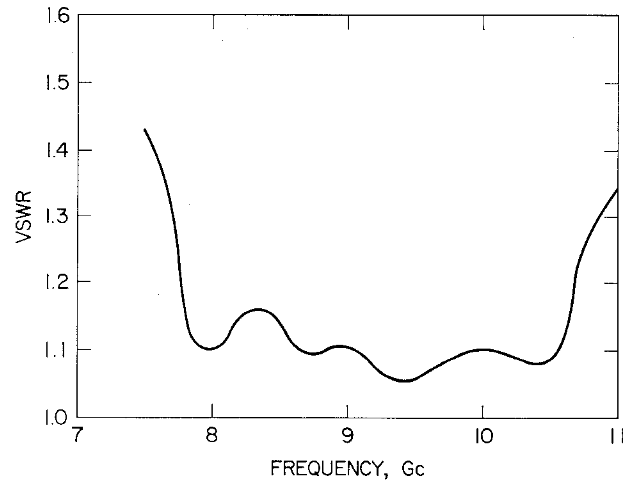

Fig. 5—Typical VSWR vs frequency for transition from an RG-52 to an alumina-filled waveguide.

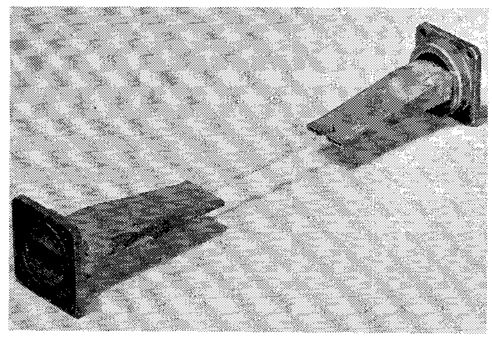

Fig. 6—Transition from an RG-52 to a titania-filled waveguide.

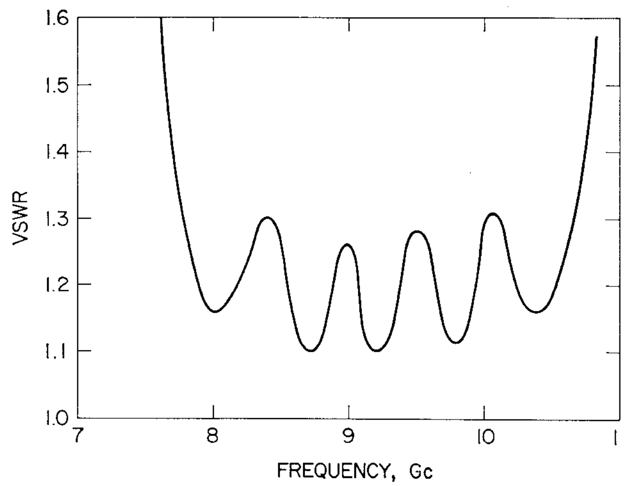

Fig. 7—VSWR vs frequency for transition from an RG-52 waveguide to a titania-filled waveguide.

TABLE I

EXPERIMENTAL RESULTS

Frequency Range, kMc	Dielectric Material	Dielectric Constant	Maximum VSWR	Maximum Loss, db
7.0– 9.0	alumina	9.4	1.16	0.1
8.0–10.6	alumina	9.4	1.16	0.1
7.8–10.7	titania	100.0	1.3	0.4 (includes length of guide)
22.0–25.0	alumina	9.4	1.2	0.2
32.0–38.0	alumina	9.4	1.5	0.5

Experimental Results

The performance of the matching device depends upon the termination of the filled guide. Two termination methods were used in obtaining experimental data. With the ruby- or alumina-filled guides, we used a termination made from a polyiron-filled waveguide, which absorbs the power propagating down the waveguide. Fig. 4 shows the transition to an alumina-filled waveguide and the polyiron termination; the corresponding VSWR is shown in Fig. 5. The rutile-filled waveguide was more difficult to terminate with an absorbing material. Polyiron has an insufficiently high dielectric constant to be well matched to rutile. Titania with a 20 per cent doping of silicon carbide offers promise as an absorbing material for titania- and rutile-filled waveguides and is presently being tested.

For the test data for titania reported here, a second method of terminating the filled waveguide was used. In this method, two transitions are used—one to match into the filled waveguide, and one to match out to the air-filled waveguide and then to a standard termination (see Fig. 6). The titania-filled waveguide is 0.040 in in the b dimension and 0.090 in in the a dimension. The VSWR plot for this configuration is shown in Fig. 7. Since the measured reflected power is from both transitions, the data in Fig. 7 were those calculated for each transition. Performance data for some typical transitions appear in Table I.

Conclusion

It has been shown that a broad-band RF impedance match from air-filled to dielectric-filled waveguide is possible using an abrupt transition with an appropriate susceptance match. The technique described offers considerable improvement in performance over more conventional dielectric taper transitions, and is easier to fabricate. Immediate application is seen in maser circuits where the waveguide is filled with high dielectric maser material.

Excitation of Plasma Waves in an Unbounded Homogeneous Plasma by a Line Source*

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Summary—**The radiation characteristics of a line source of magnetic current embedded in a homogeneous electron plasma of infinite extent are investigated for the case in which a uniform magnetic field is impressed externally throughout the medium in the direction of the source. The single-fluid theory of magnetohydrodynamics is employed. A very simple model is assumed for the plasma. Under this assumption, it is found that there are two modes of propagation of waves of small amplitude. By examining the behavior of these modes in the limiting cases of vanishing external magnetic field or infinite source frequency, they are identifiable as the modified forms of the usual plasma and optical modes which exist in an isotropic electron plasma. The dispersion relations for these two modes are discussed. The power radiated in each of the two modes is also evaluated. It is found that the power radiated in the optical mode is always lower than that due to the line source in free space, whereas the power radiated in the plasma mode is higher than that value for certain ranges of the source frequency.**

Introduction

THE STUDY OF the radiation characteristics of localized electromagnetic sources in an unbounded ionized gaseous medium, known generally as plasma, has application to the problem of radio communication with missiles at the time of their re-entry into the earth's atmosphere and with space vehicles passing through the ionosphere and other ionized regions in interplanetary space. In recent years, this subject has received considerable attention in literature. Previous investigations of this subject may be conveniently grouped into three categories.

In the first category, the plasma is assumed to be incompressible so that the presence of the longitudinal plasma waves is ruled out. Under this assumption, the plasma reduces to a dielectric medium characterized by a tensor dielectric constant. In the absence of an external static magnetic field, the tensor dielectric constant becomes a scalar. The characteristics of plane wave propagation in such an anisotropic dielectric medium have been studied, but without taking into account the sources which excite these waves. Also, the radiation characteristics of sources in a plasma idealized by an anisotropic dielectric medium were investigated. For example, Arbel[1] has treated the problem of radiation from a point source in an incompressible homogeneous plasma medium of infinite extent.

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† Gordon McKay Laboratory, Harvard University, Cambridge, Mass. On leave from Defence Electronics Research Laboratory, Hyderabad, India.

[1] E. Arbel, "Radiation from a Point Source in an Anisotropic Medium," Polytechnic Inst. of Brooklyn, N. Y., Res. Rept. No. PIBMRI-861-60; November, 1960.